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ABSTRACT

Cognitive development and various educational implications are discussed in terms of Donald Saari's model of the interaction of a learner and the enviroment and the constraints imposed by the inefficiency of the learner's cognitive system. Saari proposed a hierarchical system of cognitive structures such that the relationships between structures at one level form the structures of the next level. Perceptual identification is the lowest level with simple set formation included in the next level. The third level contains structures corresponding to Piaget's formal operational structures. The learner interprets events on the basis of the structures in his perception of the environment. Each structure has a magnitude that reflects its success in interpreting past events. Interpreting events is assimilation; changing structures is accommodation and learning. Assimilation takes in only a subset of the environment and is not instantaneous. These inefficiencies are discussed in relationship to various aspects of teaching and learning and individual differences. The concept of separation of stimuli is discussed in relation with Witkin's cognitive style dimension of field-dependence-independence and Dunker's functional fixedness. (Author/CTM)

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Some Instructional Implications from a Mathematical Model of Cognitive Development*

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A model of human behavior should serve a variety of purposes. Primarily, such a model should be predictive; that is, it should describe the nature of human behavior, given a particular setting. In addition, a model should facilitate communication among researchers and, if general enough, should aid in the synthesis of results from the various subfields of human cognition--memory, learning, perception, etc. One possible tool for the development of such a model is mathematics.

Although mathematical models have been developed for a variety of physical phenomena, mathematics has not been as successfully applied in modeling human behavior. This is partially due to fundamental differences between these areas of study. Whereas many physical phenomenon can be described in quantitative terms, human development often involves qualitative aspects. Hence, models of human behavior must be qualitative rather than quantitative. Recently, through the use of equilibrium theory, Donald Saari (1976, 1978) has created a mathematical model of adaptation which seems to reflect the qualitative and dynamic aspects of cognitive development.

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The Saari model describes an organism, or learner, that is an active interpreter of his environment. The learner creates a model world representation of the real world. The model world is the learner's perceived structure on the environment, which may or may not be consistent with its actual structure. This perceived structure determines the learner's interpretations of environmental events. In turn, environmental events induce changes in the structure the learner perceives; that is, in his model world. Learning and cognitive development are thus described in terms of a dynamic interaction between the learner and the environment.

Because learning is described in terms of the learner's interaction with the environment, the Saari model is especially applicable to education. In this model, what is learned is related to the efficiency of the learner's cognitive system compared to the structure of the environment. From this characterization, it may be possible to describe methods of improving instruction based on the restructuring of the learner's environment, and thus provide guidelines for designing instructional environments that maximize learning by all pupils.

As yet, it is not clear whether the Saari model is an accurate representation of cognitive behavior. The Saari model describes a learner, or organism, whose behavior is based on an adaptive system. The basic question is whether the behavior of this organism mirrors human behavior; if so, then the Saari system can be used as a model of human cognition. Initial comparisons of the model learner's behavior to human behavior indicate promising results. The behavior of the Saari model organism has many of the characteristics of human behavior that Piaget has noted--task specificity, décalage, centering, --

and also exhibits qualitatively different stages of development. In addition, other behaviors seem to conform to observed human responses such as the eye-witness phenomenon and latent learning. Yet, much additional work remains to be done in validating this model.

This paper will focus on the aspects of the Saari model that are most immediately relevant to education--the constraints imposed on the environment by the inefficiency of the learner's cognitive system. Selected conclusions about learning will then be compared to established empirical results in an attempt to assess the compatibility of the Saari model organism's behavior and observed human behavior.

At this time, space does not permit a complete presentation of the model. However, a short synopsis provides a sufficient basis for the present discussion. A complete description of the model is available in (Saari, 1978).

The Saari model presents a learner who constructs a mental model world, which is in essence the learner's interpretation of the real world. The model world is made up of cognitive structures. These structures are hierarchical in that relationships between structures at one level form the structures of the next level. For our purposes, we will consider the lowest level to be that of perceptual identification, (e.g., soft, hot, etc.) with simple set formation and implication included as elements of the next level (α). Note that level α includes Piagetian pre-concrete operational structures such as less than and greater than, but these have not yet been combined to form the relation $A < B \rightarrow B > A$. The next level, β, coordinates α level structures. Piaget's concrete operational structures are a subset of this level. The following level, γ, then consists of relations between the β level structures and hence, contains structures corresponding to Piaget's formal operational structures.

Each higher level structure is "built" on lower order ones, which means the use of any particular higher order structures is tied to the particular lower order structure involved. Therefore the use of higher order structures is task dependent. This means that the learner may use a particular higher order relationship in one situation, but not use it in a different situation though it may still be appropriate. This aspect of behavior has been noted empirically by Piaget, among others, and termed décalage.

The organism bases his interpretation of events in the environment on these model world structures. The nature of the available structures determines the way an organism will interpret environmental stimuli. Each cognitive structure has a magnitude that reflects the relative strength or "fitness" of that identification or relation. The "fitness" of a structure can be thought of as a reflection of the success that it has had in interpreting past events. Then, structures with higher magnitudes are more likely to be used as interpretations of new environmental events than those with smaller values. Note that this is rather cyclic: structures that are more fit tend to be used as interpretations. If the structure proves to be an adequate interpretation, it becomes more fit, and thus, is more likely to be used as an interpretation in the future. In terms of human behavior, this type of cycle describes habit and habitual actions.

The process of interpreting environmental events is assimilation. Assimilation assigns an event (environmental stimulus) to a particular cognitive structure. Assimilation is most easily thought of as a searching process seeking the structure that provides the best interpretation of the event. However, assimilation is also influenced by the magnitude of the various structures. Assimilation is attracted to structures with larger values; that is, those that

are the most fit. A useful analogy (Saari, personal communication) is to consider the cognitive structures of differing magnitudes as magnets of varying strength. Each magnet is selective with respect to the type of material it can attract, reflecting the limited applicability of any particular cognitive structure. Assimilation is then attracted to the largest valued structure which provides a possible interpretation for the event. (Analogously, the strongest magnet that attracts that material.)

The existing structures may not always provide adequate interpretations of the environmental events. When this happens, the cognitive structures change in an attempt to provide a more adequate interpretation. The process by which the model world structures change is accommodation. Accommodation is an adjustment of the "best" structure or structures selected by assimilation to improve the interpretation. For example, accommodation can be a change that appears to expand an existing structure (e.g., extention of the concept "triangle" from three sided polygons with acute angles to three-sided polygons; that is, from one restricted to this △, to now include this ▷), or that adjusts a structure in response to the selection of two adequate interpretations by assimilation (e.g., interpreting □ as both a square and a rectangle). Note that accomodation is directly dependent upon the initial interpretation (s) selected by assimilation.

In terms of this model, learning is a change in the cognitive structures, and as such, is part of accommodation. In other words, learning is the development of new relations between existing interpretations. This model states that it is "the given environmental structures which provide the impetus for the creation of these relationships between interpretations..."(Saari, 1978, p.61).

To avoid having an omniscient organism, assimilation is required to take

in a subset of the environment, rather than focusing on just a single stimulus or event. Assimilation then searches the cognitive structures for the "most fit" interpretation. Assimilation is described as an adjustment procedure and, as such, is not assumed to be instantaneous. Instead, assimilation requires some time interval to arrive at an interpretation. Consequently, all the events in this time span are considered to be part of the same environmental subset. Other restrictions on the subset stem from the physical limitations of the organism's perceptual systems. These restrictions are the size of the perceptual field, the degree to which stimuli in the same environmental subset are "separated" (perceived as separate entities), and perceived sequencing of events in the subset. Overall then, the process of assimilation is inefficient in the following ways: 1) interpretation is not instantaneous, hence the environmental subset has a temporal bound; 2) the perceptual system imposes limitations on the "field of view" in terms of size of perceptual field, separating distinct stimuli, and sequencing of stimuli. This implies that an event is not interpretated in isolation. Instead, a subset of the environment consisting of a perceptual and temporal region about the stimulus is interpreted.

It seems reasonable to assume that the efficiency of these processes varies across individuals. These differing levels of efficiency can be used to describe various "cognitive styles" and to predict behavior for learners with a particular style or combination of styles. This provides some educational implications, and with them, an opportunity to compare the model learner's behavior with corresponding empirical results. Hence, we will be able to partially investigate the validity of some aspects of the Saari model.

To start, consider the inefficiency of assimilation which has several

implications for education. The first stems from the stipulation that the determination of a suitable interpretation takes time, especially when there is no "outstandingly fit" structure for assimilation to use. When applied to a classroom setting involving teacher-student interaction, this implies that in posing questions, a teacher must provide enough response time to allow the students' assimilation processes to find an appropriate interpretation. If too little time is given many students will not have determined a suitable interpretation, due to the inefficiency of their assimilation process. As a result few students would be expected to respond, and at least some of the answers would not be as good as they might have been had the students had more time. Furthermore, one would expect the same students (those with more efficient assimilation processes) to respond more often and, overall, have more success in oral participation due to the agreement between the efficiency of their assimilation process and the pace of the environment in which they are operating. (Note, however, that although efficiency of assimilation is a factor affecting student response time, it is not the only factor; student response time depends on the availability of a "fit" structure as well.) At the opposite extreme, too much response time or delay between questions would also be determental, especially in situations in which a series of questions was meant to lead to a specific conclusion. Students could fail to see the questions as being related, and instead, view them as separate events. Overall, it seems plausible that various class tempos favor particular students: students with faster assimilation processes would do better in a faster paced environment, while students with slower processes would benefit from one that went at a slower pace.

Many classroom situations support the above implications. That the same group of students almost always responds first is not unusual for most classrooms. Research on the effects of the amount of time teachers waited for student response on student performance provides further support for the contentions made above. Rowe (1978) found that when teachers increased their wait-times (to an average of three seconds from the usual one second or less), students gave longer and more appropriate responses, student's confidence in their answers increased, failure of students to respond decreased, and students asked more questions. Although no research has been located that speaks to the opposite case, the effects of an extreme amount of wait-time seem to be fairly self-evident.

The efficiency of a learner's assimilation process can also effect his ability to understand causality and to form implications. A very efficient assimilation process requires little time to find an interpretation, and so, includes a very small temporal region of the environment in the subset that is interpreted. This means that organisms with very efficient assimilation processes may have difficulty forming some causal relationships because the cause and effect would be interpreted as parts of separate environmental subsets rather than as parts of the same subset. On the other hand, an extremely inefficient assimilation process needs a longer time to select an interpretation, meaning that a large temporal region of the environment is being considered. Thus, an individual with such an assimilation process would have so much in an environmental subset that it would be very difficult to isolate events, which consequently would hinder the formation of correct implications. Hence, there seems to be an optimal "band of efficiency"

between these two extremes. However, caution must be exercised in attempting to make inferences about an individual's intellectual capabilities based on his temporal efficiency. One of the major implications from the Saari model is that "there is an optimal region of efficiency for the process of assimilation which is determined by the given structure of the environment" (p.65). In other words, the "best" speed for assimilation depends on the situation at hand. Therefore, the bounds of the efficiency band are determined by the particular causal relationship to be formed and the context in which it occurs.

These differences in the efficiency of assimilation should be considered when attempting to teach children implications or causal relationships. Potential problems resulting from overly efficient or inefficient assimilation can be avoided by compensating for its shortcomings through the structuring of the learning environment. Individuals with small temporal regions require events with short temporal spans; therefore, the Saari model implies that a faster paced environment, or a more "busy" classroom, should be optimal for learning by these students. On the other hand, learners with large temporal regions tend to have trouble isolating important stimuli due to the large amount of information in each of ther environmental subsets. To compensate for this, emphasis must be placed on the stimuli that are part of the relation to be developed. Emphasis can be added through verbal instructions (e.g., draw attention to particular stimuli) and/or, by removing extraneous stimuli, if possible. More sterile environments contain fewer stimuli, and hence, would make it easier for slower assimilators to focus on the aspects of the situation relevant to the development of the desired implication.

Note that the Saari model implies that students with different degrees of

assimilation efficiency require different environmental structuring. Individuals whose assimilation efficiency falls outside the "band of efficiency" for a particular relation will not all benefit from the same type of correction attempts. Since these are the people who would be expected to have learning problems in certain areas, it follows that the same type of instructional remediation will not be adequate for all pupils. Instead, remediation steps must be adjusted to fit the degree of efficiency of the learner's assimilation process.

The methods for modifying the effects of assimilation efficiency mentioned above have been based on adjustments of the environment, rather than on manipulation of the efficiency of assimilation itself. This is because the model does not, as yet, include the possiblity of conscious control of assimilation; that is, the ability to slow down or speed up the assimilation process at will. Instead, assimilation is effected by the amount of information in the environmental subset to be interpreted. The information perceived can be manipulated by the actions of someone other than the assimilator(e.g., directing the assimilator's attention, or structuring the environment to emphasize particular features),or perhaps even altered by the assimilator himself (e.g., self-instruction in the use of particular visual scanning strategies). At this point, it is unclear whether the dimension of conscious control must be included in the model for it to be an accurate representation of human behavior. Since it is not part of the model at this time, we have not considered it in the above analysis, but realize it is an assumption that may have to be added to this model in the future to provide an accurate description of human behavior.

Efficiency of assimilation raises two additional questions: 1) does it vary across situations for a single individual at a particular level of development, and 2) it is developmentally stable; that is, is it constant for an individual as he develops? The answer to the first questions is that the speed of assimilation (i.e., the rate at which it searches for an interpretation) should generally remain the same, though its efficiency in terms of time to select an interpretation may vary, depending on the complexity of the perceived environmental event and its familiarity. If the environmental subset is very rich in stimuli, assimilation may have more possible interpretations to search through. In contrast, if the event is very familar, an interpretation is easily selected due to the "fitness" this structure has developed through past experience.

Before considering the second question, cognitive development must be defined in terms of the Saari model. Recall that learning is the creation of new cognitive structures and/or the modification of existing ones. A younger individual would then be expected to have fewer structures than a more mature learner. This implies that as an individual develops, the "absolute" efficiency (measured in terms of response time) of his assimilation process will decrease (i.e., response time will increase), due to the increased number of structures it must search through. However, this trend holds only for relatively novel situations. In very familar situations, the assimilation efficiency should appear to increase, due to the existence of a "fit" structure with which to interpret the event. A fit structure has a large magnitude, so assimilation could be expected to be "attracted" to this structure fairly rapidly. Further-

more, the relative efficiency of assimilation should remain constant for individuals with identical cognitive structures. However, it seems that individuals could learn to compensate for an overly efficient assimilation process by attempting to 1) slow it down, or 2) widen their perceptual scope. As mentioned earlier, the first is not included within the present framework of the Saari model. The second type of action would increase the amount of information to be interpreted, and so could alter an individual's apparent assimilation efficiency. Therefore, because the novelty or familiarity of the environmental events and the development of compensation behaviors can effect the apparent efficiency of assimilation, these factors must be taken into consideration in searching for developmental trends.

Besides the temporal efficiency of assimilation, the Saari model organism also has other limitations imposed on it: the size of the perceptual field, the degree to which stimuli are seen as distinct, and the perceived sequencing of stimuli. Only the second of these will be considered here.

Separation of environmental stimuli has two different interpretations. The first concerns the physical capabilities of perception--how small an amount of stimulus separation can be perceived, either in terms of physical or temporal separation (e.g., how fast can a series of still photographs be shown and still be perceived as separate?) and will not concern us here. The second, however, is of interest. This is more of a separation of stimuli based on assimilation. This separation is the extent to which environmental stimuli that are obviously perceptually distinguishable are viewed as separate. One facet of this is the degree to which stimuli are interpreted as independent of other environmental stimuli, or are viewed "in context". In other words, to what extent are interpretations of stimuli influenced by their surroundings? Note that this aspect

of the Saari model's separation dimension appears to be identical to the cognitve style dimension of field-dependence-independence, which is defined as "the extent to which the person perceives part of a field as discrete from the surrounding field as a whole, rather than embedded in the field" (Witkin, Moore, Goodenough, & Cox, 1977, p. 7).

Variations in the degree to which environmental stimuli are separated will effect a person's interpretation of a situation. This is most evident in problems in which the solution requires a restructuring of the original situation. An element (s) of the problem must be removed from the context in which it is presented and used in a different way to reach a solution. Examples of such problems are the tasks used by Dunker (1945) to investigate functional fixedness.* Because individuals who separate stimuli to a low degree would have more difficulty removing an element from the given context and using it in another way than those who separate events to a greater extent, they should have more trouble solving this type of problem.

Observed human behavior seems to verify this prediction. Because field-dependence-independence is included in the Saari separation dimension, research on the performances of field dependent-independent individuals can be used to investigate the validity of the Saari model implications about behavior in the type of problem solving situation mentioned above. And in fact, research does seem to support these contentions. Witkin et al. cite an unpublished study by Frances Harris in which he found that "to a striking degree, people who were field-independent (as determined by laboratory tests) more easily overcame the

* One of these is the "pliers problem" in which a person is asked to construct a shelf consisting of a board resting on two supports and is given a number of objects including a board, with one support attached, and a pair of pliers. The support is nailed to the board in such a way that it must be detached from the board (by removing the nail with the pliers) before it can be used in the desired way. The solution requires the person to then use the pliers as the other support for the board.

predominant context" (Witkin et al., 1977, p. 8) in Dunker's "pliers problem".

Here, using only two of the cognitive inefficiences from the Saari model--temporal and separation--we have been able to make influences about several diverse aspects of education: classroom tempo and student response, students' formation of implications and problem solving behavior in certain situations. Other inefficiences and constraints not explored here, such as environmental sequencing of recognized events, repetition (to develop fit structures) and spatial proximity provide further implications. For example, Saari (1978) states "the spatial proximity requirement suggests that there is a size and spacing constraint imposed on objects. That is, a sequencing may not be noticed if objects are too large or too far apart" (p. 67). Similar conclusions were reached by Schultz (1977) in a study of childrens understanding of geometric transformations, thus providing empirical support for this implication.

However, a number of significant questions are, as yet, unanswered. Can the cognitive inefficiencies mentioned in the Saari model be used to describe various learning difficulties? What are the effects of various combinations of inefficiences? Does the Saari separation dimension really encompass field-dependence-independence? Can the Saari inefficiencies be used to describe other dimensions of cognitive style, such as reflection-impulsivity?

Although the Saari model does give two rather novel results--the idea of a "band of efficiency" and the notion that performance depends on the fit between the learner's assimilation efficiency and the structure of the environment--most of the results mentioned here were not completely new, as evidenced by the existence of empirical support. What is very surprising though, is that the same model provides the suggestions made here about student response

time, the formation of implications and problem solving behavior, as well as those made by Saari regarding the eye witness phenomena, latent learning, and cognitive development. Thus it seems that the Saari model may provide a general discription of human behavior and as such could be a very important source of educational implications.

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